Mean Angle

To calculate the mean angle ($\bar{\theta}$), we first convert all our angles from degrees to radians:

$$\theta_{radians} = \theta_{degrees} \times \frac{\pi}{180}$$

Next, we calculate the average cosine (X) and sine (Y) for each angle:

$$X = \frac{1}{n}\sum_{i=1}^{n} \cos \theta_i$$

$$Y = \frac{1}{n}\sum_{i=1}^{n} \sin \theta_i$$

Then, we calculate the mean angle ($\bar{\theta}$) using a slightly different correction depending on the quadrant of the mean:

- If X and Y are both positive, then

$$\bar{\theta} = \tan^{-1}\left(\frac{Y}{X}\right)$$

- If X is positive and Y is negative, then

$$\bar{\theta} = 2\pi - \tan^{-1}\left(\frac{Y}{X}\right)$$

- If X is negative and Y is positive, then

$$\bar{\theta} = \pi - \tan^{-1}\left(\frac{Y}{X}\right)$$

- If X and Y are both negative, then

$$\bar{\theta} = \pi + \tan^{-1}\left(\frac{Y}{X}\right)$$

Now you can convert $\bar{\theta}$ from radians back to degrees:

$$\bar{\theta}_{degrees} = \bar{\theta}_{radians} \times \frac{180}{\pi}$$

Rayleigh's Test

Rayleigh's Z test will determine whether a set of angles is random ($p > 0.05$) or has a preferred direction (oriented, $p < 0.05$).
We start by calculating X and Y the same as above for the mean angle. Then, we calculate the length of the mean vector (r):

$$r = \sqrt{X^2 + Y^2}$$

r can be between zero and 1, zero being all angles are equally distributed around the circle, and 1 being all the angles are exactly the same.

Next, we calculate Rayleigh's test statistic z using the sample size n:

$$z = nr^2$$

Finally, use the look-up table on the next page to find whether or not our z is larger than the critical value for significance. Find the row that corresponds to n and the column the corresponds with a probability of 0.05. If your z is greater than this value in the table, then your p is less than 0.05 and you reject randomness and conclude the samples have a preferred direction.

Critical z Values for the Rayleigh's Test

Taken from Zar, 1981 Table B.32

n	α: 0.50	0.20	0.10	0.05	0.02	0.01	0.005	0.002	0.001
6	0.734	1.639	2.274	2.865	3.576	4.058	4.491	4.985	5.297
7	0.727	1.634	2.278	2.885	3.627	4.143	4.617	5.181	5.556
8	0.723	1.631	2.281	2.899	3.665	4.205	4.710	5.322	5.743
9	0.719	1.628	2.283	2.910	3.694	4.252	4.780	5.430	5.885
10	0.717	1.626	2.285	2.919	3.716	4.289	4.835	5.514	5.996
11	0.715	1.625	2.287	2.926	3.735	4.319	4.879	5.582	6.085
12	0.713	1.623	2.288	2.932	3.750	4.344	4.916	5.638	6.158
13	0.711	1.622	2.289	2.937	3.763	4.365	4.947	5.685	6.219
14	0.710	1.621	2.290	2.941	3.774	4.383	4.973	5.725	6.271
15	0.709	1.620	2.291	2.945	3.784	4.398	4.996	5.759	6.316
16	0.708	1.620	2.292	2.948	3.792	4.412	5.015	5.789	6.354
17	0.707	1.619	2.292	2.951	3.799	4.423	5.033	5.815	6.388
18	0.706	1.619	2.293	2.954	3.806	4.434	5.048	5.838	6.418
19	0.705	1.618	2.293	2.956	3.811	4.443	5.061	5.858	6.445
20	0.705	1.618	2.294	2.958	3.816	4.451	5.074	5.877	6.469
21	0.704	1.617	2.294	2.960	3.821	4.459	5.085	5.893	6.491
22	0.704	1.617	2.295	2.961	3.825	4.466	5.095	5.908	6.510
23	0.703	1.616	2.295	2.963	3.829	4.472	5.104	5.922	6.528
24	0.703	1.616	2.295	2.964	3.833	4.478	5.112	5.935	6.544
25	0.702	1.616	2.296	2.966	3.836	4.483	5.120	5.946	6.559
26	0.702	1.616	2.296	2.967	3.839	4.488	5.127	5.957	6.573
27	0.702	1.615	2.296	2.968	3.842	4.492	5.133	5.966	6.586
28	0.701	1.615	2.296	2.969	3.844	4.496	5.139	5.975	6.598
29	0.701	1.615	2.297	2.970	3.847	4.500	5.145	5.984	6.609
30	0.701	1.615	2.297	2.971	3.849	4.504	5.150	5.992	6.619
32	0.700	1.614	2.297	2.972	3.853	4.510	5.159	6.006	6.637
34	0.700	1.614	2.297	2.974	3.856	4.516	5.168	6.018	6.654
36	0.700	1.614	2.298	2.975	3.859	4.521	5.175	6.030	6.668
38	0.699	1.614	2.298	2.976	3.862	4.525	5.182	6.039	6.681
40	0.699	1.613	2.298	2.977	3.865	4.529	5.188	6.048	6.692
42	0.699	1.613	2.298	2.978	3.867	4.533	5.193	6.056	6.703
44	0.698	1.613	2.299	2.979	3.869	4.536	5.198	6.064	6.712
46	0.698	1.613	2.299	2.979	3.871	4.539	5.202	6.070	6.721
48	0.698	1.613	2.299	2.980	3.873	4.542	5.206	6.076	6.729
50	0.698	1.613	2.299	2.981	3.874	4.545	5.210	6.082	6.736
55	0.697	1.612	2.299	2.982	3.878	4.550	5.218	6.094	6.752
60	0.697	1.612	2.300	2.983	3.881	4.555	5.225	6.104	6.765
65	0.697	1.612	2.300	2.984	3.883	4.559	5.231	6.113	6.776
70	0.696	1.612	2.300	2.985	3.885	4.562	5.235	6.120	6.786
75	0.696	1.612	2.300	2.986	3.887	4.565	5.240	6.127	6.794
80	0.696	1.611	2.300	2.986	3.889	4.567	5.243	6.132	6.801
90	0.696	1.611	2.301	2.987	3.891	4.572	5.249	6.141	6.813
100	0.695	1.611	2.301	2.988	3.893	4.575	5.254	6.149	6.822
120	0.695	1.611	2.301	2.990	3.896	4.580	5.262	6.160	6.837
140	0.695	1.611	2.301	2.990	3.899	4.584	5.267	6.168	6.847
160	0.695	1.610	2.301	2.991	3.900	4.586	5.271	6.174	6.855
180	0.694	1.610	2.302	2.992	3.902	4.588	5.274	6.178	6.861
200	0.694	1.610	2.302	2.992	3.903	4.590	5.276	6.182	6.865
300	0.694	1.610	2.302	2.993	3.906	4.595	5.284	6.193	6.879
500	0.694	1.610	2.302	2.994	3.908	4.599	5.290	6.201	6.891
∞	0.6931	1.6094	2.3026	2.9957	3.9120	4.6052	5.2983	6.2146	6.9078

Watson's Two-sample Test

This test will compare whether or not groups of angles are different from each other, for example, Control vs. Pulsed groups.
To start, we will call the two groups G_1 and G_2 with samples sizes n_1 and n_2.
Next, make a table with 5 columns, and list all the angles from smallest to largest. Here is an example:

Angle	Group	a	b	d
15				
20				
23				
52				
65				
160				
176				
191				
217				
280				

Then, in the second column assign the group name, G_1 or G_2, to each angle:

Angle	Group	a	b	d
15	G_1			
20	G_1			
23	G_1			
52	G_1			
65	G_2			
160	G_1			
176	G_2			
191	G_2			
217	G_2			
280	G_2			

Next, the third column, column "a" gets a bit tricky. For the first row, count the number of "G_1" in the first row of column 2. For the second row, count the number of "G_1" in rows 1 and 2 of the second column. For the third row, count the number of "G_1" in rows 1-3…. Continue until the column is completed.

Angle	Group	a	b	d
15	G_1	1		
20	G_1	2		
23	G_1	3		
52	G_1	4		
65	G_2	4		
160	G_1	5		
176	G_2	5		
191	G_2	5		
217	G_2	5		
280	G_2	5		

Then, repeat the same procedure for the next column, column "b", but count the number of "G_2" instead.

Angle	Group	a	b	d
15	G_1	1	0	
20	G_1	2	0	
23	G_1	3	0	
52	G_1	4	0	
65	G_2	4	1	
160	G_1	5	1	
176	G_2	5	2	
191	G_2	5	3	
217	G_2	5	4	
280	G_2	5	5	

For each item in the last column, column "d", enter the result from the equation:

$$d = \frac{b}{n_2} - \frac{a}{n_1}$$

where "b" is the value from column "b" for that row, and "a" from column "a" for that row:

Angle	Group	a	b	d
15	G_1	1	0	-0.2
20	G_1	2	0	-0.4
23	G_1	3	0	-0.6
52	G_1	4	0	-0.8
65	G_2	4	1	-0.6
160	G_1	5	1	-0.8
176	G_2	5	2	-0.6
191	G_2	5	3	-0.4
217	G_2	5	4	-0.2
280	G_2	5	5	0

Calculate the mean of the column "d" and the variance:

$$\bar{d} = \frac{1}{n_1 + n_2} \sum d = -0.46$$

$$v = \sum \left(d - \bar{d}\right)^2 = 0.644$$

Finally, we can calculate our test statistic:

$$U^2 = \frac{n_1 n_2 v}{(n_1 + n_2)^2} = 0.161$$

If $U^2 > 0.187$ then the two groups are significantly different ($p < 0.05$), however, if $U^2 < 0.187$ then the two groups do not differ.